Traveling waves induced by parameter fluctuations in rings of coupled maps

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Abstract

We describe a novel mechanism for inducing traveling-wave attractors in rings of coupled maps. Traveling waves are easily produced when parameters controlling local dynamics vary from site to site. We also present some statistical results regarding the distribution of periodic time-evolutions. © 2001 Elsevier Science B.V. All rights reserved.

Wave activities and traveling waves are a common and rather crucial atmospheric phenomena. They are associated with phenomena in the ocean–atmosphere system [1–3], namely the well-known interannual oscillations such as the El-Niño-Southern Oscillation and the North Atlantic Oscillation [4–6]. Wave dynamics underlies atmospheric evolution ruled by a number of collective phenomena and depends on global interchanges among natural oscillations mediated by traveling waves. Thus, it is important to understand the subtleties of traveling wave generation to improve our ability of forecasting long-term behaviors of the atmosphere (climatic changes).

Traveling waves in rings of diffusively coupled maps were observed by Kaneko [7,8] while varying initial conditions in a lattice composed by *identical* oscillators (homogeneous lattice). The detailed structure of parameter space and the dynamical characteristics of wave propagation in homogeneous lattices were described recently by

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us [9,10]. However, for meteorological purposes a more realistic hypothesis is obviously to allow for parameter fluctuations along the ring (heterogeneous lattice). The main point here is to report that traveling waves may be easily produced by varying the parameters controlling local oscillators.

To this end, we consider a diffusive ring of coupled maps with time-evolution updated in parallel according to $x_{t+1}(i) = f(x_t(i)) - \varepsilon \varphi(t, i)$, where $\varphi(t, i) = f_i(x_t(i)) - [f_{i-1}(x_t(i-1)) + f_{i+1}(x_t(i+1))]/2$ represents the feedback from the nearby environment and ε is the coupling strength between neighbors. The local oscillators are $f_i(x) = 1 - a_i x^2$, where a_i is chosen randomly and uniformly in the interval $I(a^*, \delta) = [a^* - \delta/2, a^* + \delta/2]$, with $i = 1, \ldots, L$, L being the lattice size. A ring is obtained by enforcing periodic boundary conditions, i.e., $x_{L+1} = x_1$. In this model, the homogeneous limit is attained by taking $\delta = 0$. We will fix the set of initial conditions in the ring, $\{x_0(i)\}$, chosen randomly from a uniform distribution in $[-1, 1]$, and investigate the time evolution of the patterns formed when varying the distributions $\{a_i\}$ in the interval $I(a^*, \delta)$.

As discussed elsewhere [9,10], the time-evolution of patterns may be characterized by five *classes*, three periodic (**S**tatic, **P**ositively or **N**egatively moving) and two aperiodic (**H**esitating and **C**haotic). Illustrative examples of time-evolutions in each of these classes are shown in Fig. 1. Although this figure was obtained for a heterogeneous
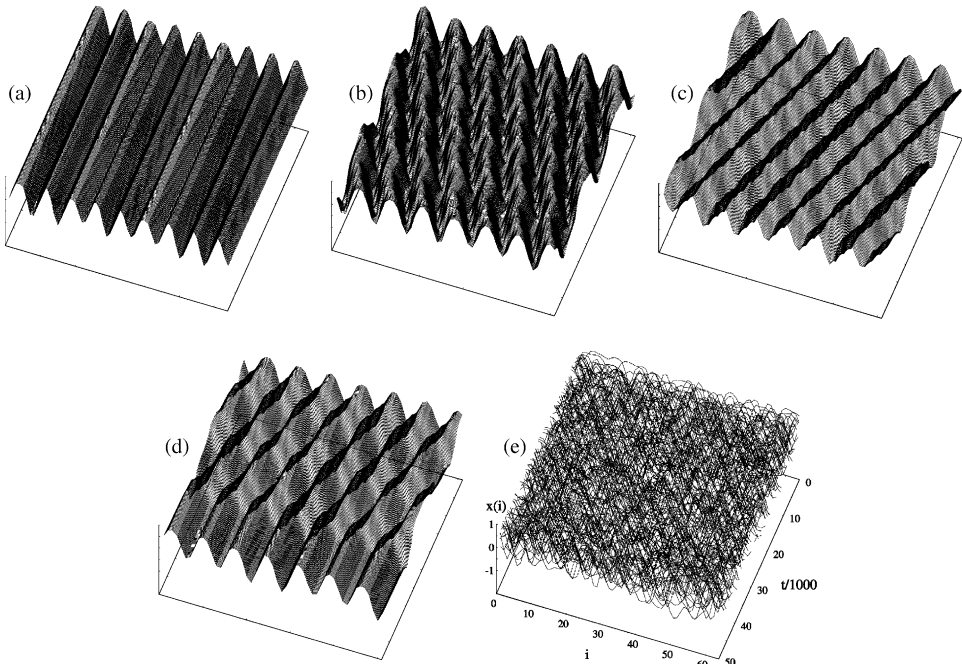

Fig. 1. Examples of modular time-evolutions found in heterogeneous rings of maps. There are three periodic classes, (a) class *S* ('static'), (b) class *P* ('positively' moving), (c) class *N* ('negatively' moving) and two aperiodic classes, (d) class *H* ('hesitating'), (e) class *C* ('chaotic'). Classes *P* and *N* consist of, respectively, positive and negative traveling waves.
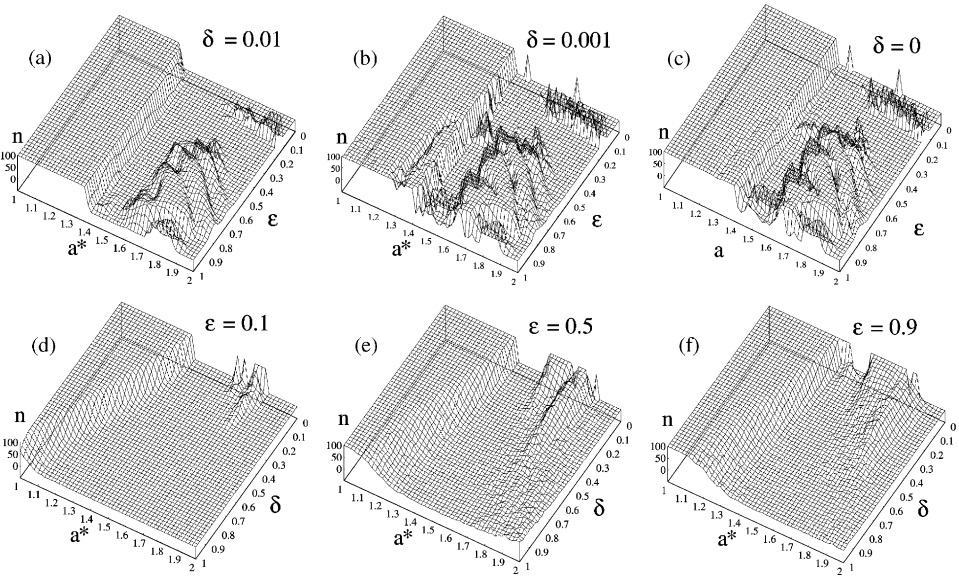
Fig. 2. The $a^* \times \varepsilon$ distribution of time-evolutions for (a–b) heterogeneous ring ($\delta \neq 0$) and (c) homogeneous ring ($\delta = 0$). The remaining figures show representative $a^* \times \delta$ distributions of the (d) weak, (e) intermediate and (f) strong coupling regimes. For $\delta = 0$, n gives the quantity of periodic time-evolutions out of 100 different initial conditions; in the remaining cases, the initial condition is *fixed* and the periodic time-evolutions are counted out of 100 different distributions of a in an interval $I(a^*, \delta)$ (see text).

lattice, it is similar to what one observes for homogeneous rings when varying initial condition [9,10].

Heterogeneity implies a more complicated parameter space: instead of the $a \times \varepsilon$ diagram of the homogeneous case, we must consider now the distributions of periodic classes in diagrams involving either $a^* \times \delta$ and $a^* \times \varepsilon$.

Fig. 2(a–c) show in detail the distribution of periodic time-evolutions in the $a^* \times \varepsilon$ plane, on a 50×50 mesh, for $\delta = 0.01, 0.001$ and 0 (homogeneous limit), while Figs. 2(d–f) show similar distributions but in the $a^* \times \delta$ plane for $\varepsilon = 0.1, 0.5$ and 0.9, all computed for $L = 64$. The first row shows that periodic time-evolutions are easily found in parameter space when varying local non-linearities. Further, the distribution of periodic time-evolutions for heterogeneous rings ($\delta \neq 0$) was obtained by varying the a-distributions, always with the *same* initial condition; surprisingly, for $\delta = 0.001$ one sees a distribution quite similar to the distribution for $\delta = 0$, obtained by varying initial conditions. Table 1 presents a summary of the features of periodic time-evolution not recognizable from Fig. 2. This summary is valid both for homogeneous and for heterogeneous rings.

Fig. 2(d–f) show typical distributions of periodic time-evolutions in the $a^* \times \delta$ plane, for fixed ε. As one sees, the absolute frequency, n, decreases with increasing δ. Furthermore, the plateau of periodicity (see Table 1) has its upper boundary, in the a^* axis, decreasing when δ increases. These features remain valid for both weak and strong coupling regimes.

Table 1
Main features of the distributions of time-evolution for *heterogeneous* rings of maps for $L = 64$. Numerical values are approximate; $a_\infty = 1.401155\ldots$ is the accumulation point of 2^ℓ cascade of the uncoupled local map. Domains containing only class S form the 'Plateau of periodicity'

$a \backslash \varepsilon$	$(0.0, 0.1)$	$[0.1, 0.2]$	$(0.2, 0.4)$	$[0.4, 1]$
$[-0.25, 1.27]$	S	S	S	S
$[1.27, a_\infty]$	S	S	S	S + Low velocity
$[a_\infty, 1.6]$	$H + C$	$H + C$	$H + C$	$H + C$
$[1.6, 1.9]$	$H + C$	$S + P + N$	$H + C$	S + High velocity
$[1.9, 2.0]$	$H + C$	$H + C$	$H + C$	$H + C$

To conclude, it is plausible to argue that generic traveling waves observed in natural phenomena may be modeled as a combination of two complementary procedures, (i) variation of the initial conditions and (ii) fluctuations of local parameters, since both reproduce observed behavior. A few points need to be further investigated. For instance, instead of working with random distributions of local parameter, a_i, we could also use random distributions of coupling strength, ε_i, to induce traveling waves or other time-evolutions. One could also consider parameter fluctuations both in space and in time. Finally, from a meteorological point of view, interesting open questions concern the reasons for the great symmetry between P and N time-evolutions and for the relatively low magnitude for the velocities. These questions are currently being investigated and will be reported in due course.

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