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Synchronization and predictability under rule 52, a cellular automaton reputedly of class 4

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Abstract

We study the complexity exhibited by the *gliders* of rule 52, a totalistic cellular automaton reputedly capable of highly intricate behaviors. Such gliders are responsible for information flow and long-range spatial correlations frequently used to classify complexity. We discover an unexpected simplicity in all computable gliders, shown to arise from simple juxtaposition between active and inactive synchronization patches linked by a remarkably small set of *communication interfaces*. We classify such interfaces and argue rule 52 to be interesting but of limited asymptotic complexity.

One of the most challenging and interesting phenomena of dynamical systems is that connected with the spontaneous generation of complex structures and patterns in space and time [1–3]. While the emergence of periodic and chaotic temporal behaviors has been studied in detail during the last two decades, the genesis of spatio-temporal regularities in extended chaotic systems with several degrees of freedom remains a much less understood topic.

A very popular way of investigating complicated spatio-temporal behaviors is by assuming their complexity to be due to cooperative effects between a number of smaller subsystems evolving under simple rules and operating on relatively few local degrees of freedom. This "reductionistic approach", which in essence asserts that complex things may be reduced or explained by simpler "more fundamental" parts, is a point of view that may be traced back to the ancient pre-Socratic Greek atomistic view of nature or, much more recently, to Descartes, who argued that, e.g., animals could be reductively explained as automata [4]. Descartes envisioned the world like a huge machine, composed of a myriad of pieces resembling clockwork mechanisms, whose collective macroscopic behavior could be understood by studying the individual components of its constituent mechanisms. This time-honored reductionistic approach has been recently hailed as a new kind of science [5]. Incidentally, it seems appropriate to mention that, in certain circumstances, the laws that describe the behavior of complex systems might be qualitatively rather different from those that govern its units [6].

Computationally, a highly efficient way of implementing the reductionistic approach is by resorting to cellular automata (CA), discrete dynamical systems capable of sustaining complex behaviors [7,8]. Over the last few years CA have received a great deal of attention, as may be easily corroborated by perusing the interesting papers and ingenious applications discussed in the almost 2000 pages of, e.g., two very recent conference proceedings [9,10]. Cellular automata are extensively studied and applied in physics, chemistry, biology, traffic engineering, computer science and other disciplines [7–16]. As mentioned, despite this activity, several fundamental questions still remain open, for instance, that concerning the precise elementary mechanisms responsible for the genesis and nature of the purported high-complexity of CA, a key element for applications such as compression and efficient storage of sound, pictures, and even whole motion pictures [17,18].

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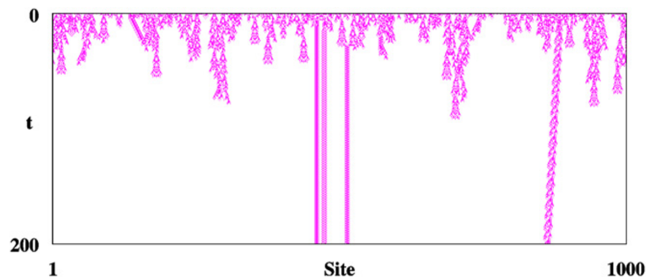

Fig. 1. Typical complex spatio-temporal patterns generated by rule 20. After a first transient (short-lived), the surviving non-quiescent activity is that of a few localized *gliders*: three stationary and time-periodic gliders located roughly at the center of the figure, and a traveling glider on the right. The collision between traveling and static gliders defines a second transient (long-lived) of the system, when the dynamics under rule 20 invariably collapses into tame class 2 behavior [24].

Some time ago, Wolfram [19,20] conjectured the possibility of collecting all spatio-temporal behaviors observed in CA into just four classes, numbered from 1 to 4. As the number of the class increases, so does the complexity of the phenomena supported by the CA contained in the class. Loosely speaking, the first three classes contain the familiar behaviors of more traditional dynamical systems, namely fixed-point, periodic orbits and chaotic behaviors. Class 4, containing all behaviors that do not properly fit into the three earlier classes, is very special and conjectured to contain automata capable of universal computations [5]. Fig. 1 illustrates class 4 behavior under rule 20, an automaton displaying the same kind of short-lived complexity seen in rule 52, the automaton that we focus in this work. While much has been discussed concerning existence and utility of class 4 automata [7,8,19–29], it is surprising that so little seems to be known about their dynamical properties.

The aim of this paper is to characterize the dynamical behavior of rule 52, perhaps the least investigated rule among the simplest candidates capable of reputedly highly complex class 4 behavior. Surprisingly, we find all computable gliders of rule 52 to present in fact relatively tame time-evolutions, consisting of simple alternations of active and inactive synchronized patches linked together by a very small set of *communication interfaces*, or motifs, where periodic activity occurs. We classify such interfaces and show how to combine them to produce gliders. Detailed statistical information about rule 52 and similar rules is presented elsewhere [30]. Before starting, we mention that while most authors find rule 52 to produce complicated behavior [19–22], it is possible to find recent work that, without entering in details, contradicts this point of view [28].

In analogy with the structures familiar from the Game of Life [7], the key elements used by Wolfram to argue for the existence of high-complexity class 4 behaviors were the so-called "gliders", illustrated in Fig. 1 for rule 20. Rule 20 means synchronous updating of the local state variable $\sigma_i(t) \in \{0, 1\}$ at site i and time t, according to the prescription

$$\sigma_i(t+1) = \begin{cases} 1, & \text{if } \Sigma = 2 \text{ or } 4, \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where

$$\Sigma \equiv \sigma_{i-2}(t) + \sigma_{i-1}(t) + \sigma_i(t) + \sigma_{i+1}(t) + \sigma_{i+2}(t).$$

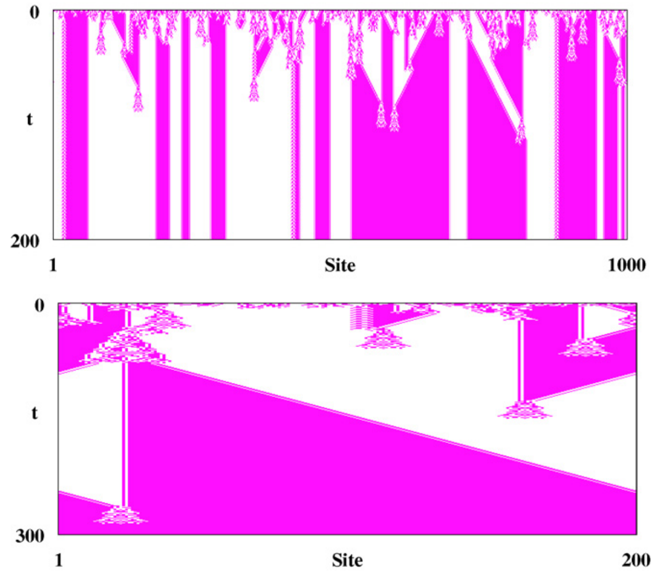

Fig. 2. Typical complex spatio-temporal patterns generated by rule 52. Top: Asymptotically, the system settles into arbitrarily large patches of synchronized activity, 0 (white) or 1 (purple, rendered in black when printed), interconnected by transition interfaces were cyclic activity occurs. Gliders may subsist *inside* individual patches but remain ephemeral as in rule 20. Bottom: Transition interfaces are static or may travel. Periodic boundary conditions eventually lead to *collisions* of interfaces and larger regions of synchronized activity. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

As clear from the above definitions, the updating of the automaton is controlled by an integer $\Sigma \in \{0, 1, 2, 3, 4, 5\}$ which is a sort of equally-weighted sampling of the local variables of 5 neighbors: the site i itself and its near and next-near neighbors. Gliders are believed to be the characteristic signatures allowing one to recognize class 4 behavior. Among all totalistic automata involving 5 neighbors, Wolfram [20] found only two rules to be of class 4, namely rules 20 and 52.

For rule 52 the updating is done according to the expression

$$\sigma_i(t+1) = \begin{cases} 1, & \text{if } \Sigma = 2, 4 \text{ or } 5, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where Σ is the same sum in Eq. (1). Now the all-*on* configuration maintains the *on* state, meaning that 4 or more adjacent *on* sites are sufficient to block information flow through them. In particular, such sites act like a rubber-band in the sense that an arbitrary number of *on* sites may be added to them without disturbing the dynamics.

Fig. 2 illustrates typical time-evolutions under rule 52 when starting from a disordered state, i.e. from a random initial condition. Here, initial states always contain an equal number of 0 and 1, and we use periodic boundary conditions, as usual. For short time scales Fig. 2 displays complexity similar to that of rule 20. The figure also shows that, again, the overwhelming tendency of such activity is to die in a relatively short time scale, with only very defined gliders surviving.

Apart from regions where the evolution resembles that of rule 20, a nice new feature of rule 52 is to contain additional domains where the evolution is *conjugate* to that of rule 20, i.e. where gliders are composed now by 0s (not by 1s) and move on backgrounds formed by 1s (not 0s). Such gliders look like photographic "negatives" of those obtained under rule 20. By experimenting with different initial conditions it is not difficult to see that behaviors that do not die after short-lived transients come in just two flavors, both present in Fig. 2. First, there are large regions of *synchronized* sites formed by homogeneous patches of either 0 or 1, represented by the two colors seen in the figure. They correspond to the binary freedom of the automata. Second, rich activity occurs in the interfaces bridging patches of different colors.

Fig. 3 summarizes the *elementary* interfaces bridging patches of different and of similar colors, respectively. In addition to

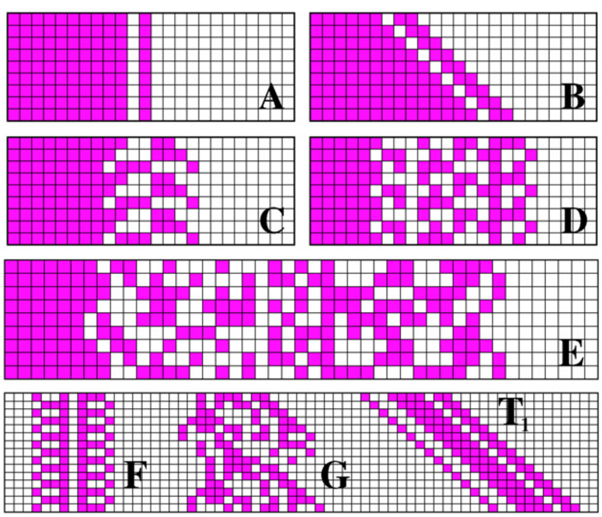

Fig. 3. The seven elementary interfaces needed to produce all asymptotic complexity computed for rule 52. Interfaces A to E bridge patches with backgrounds of different colors while F and G bridge backgrounds of the same color. In both cases, interfaces may be static or moving. The traveling glider T_1 is not elementary but a juxtaposition of B and $\overline{B}$, the conjugate of B. It travels at the "speed of light" in the system: one cell per time-step. Glider G travels at $1/9$ cell per step. Time evolves always downwards, here and subsequently. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

these elementary interfaces, rule 52 also supports the evolution of their corresponding *conjugate* interfaces, obtained by swapping simultaneously $0 \to 1$ and $1 \to 0$ in all sites of the initial conditions. For instance, the initial state ξ of the automaton A in Fig. 3 and its conjugate $\bar{\xi}$ are given by

$$\xi \equiv \ldots 11110100000\ldots, \tag{3}$$

$$\bar{\xi} = C(\xi) \equiv \ldots 00001011111\ldots, \tag{4}$$

where C denotes a conjugation operator which inverts the binary value of each cell of the automaton, and the ellipsis indicate infinite repetition of the periodic pattern preceding or following it. The elementary interfaces in Fig. 3 (and their conjugates) are the basic *atoms* which, when suitably combined, reproduce the asymptotic activity and communication boundaries illustrated in Fig. 2.

How do synchronization patches arise? Large patches of synchronized behavior arise from collisions between gliders and, as illustrated by Fig. 4, gliders arise from the different possibilities of combining elementary interfaces. For example, the leftmost structure seen in the top row in Fig. 4 may be considered either as an isolated glider propagating in the white background or, equivalently, as a double transition between backgrounds, from white → black → white, constructed with the elementary interface A and its *conjugate* $\overline{A}$, shown in Fig. 3. The inner core of glider 1, consisting of four adjacent dark cells, is the minimum one possible. Its size may be indefinitely "inflated", as hinted by the glider labeled $1'$ in the figure. Similarly, glider 2 in Fig. 4 results from an analogous double transition of backgrounds, but this time constructed with the elementary interface C and its *conjugate*, shown in Fig. 3. Its inner core may be also inflated arbitrarily, as indicated by the glider $2'$. A more complex glider is that labeled 4: it is not difficult to realize that it may be composed by first inflating glider 1 and then embedding the conjugate $\overline{F}$ of F into it. In this way we were able to produce all asymptotic structures computed for rule 52. Since "the conjugate of a fat glider is also a legal fat glider", moving in a conjugate background, one sees that the few structures shown in Fig. 4 allow one to easily recognize new gliders by the simple expedient of consid-

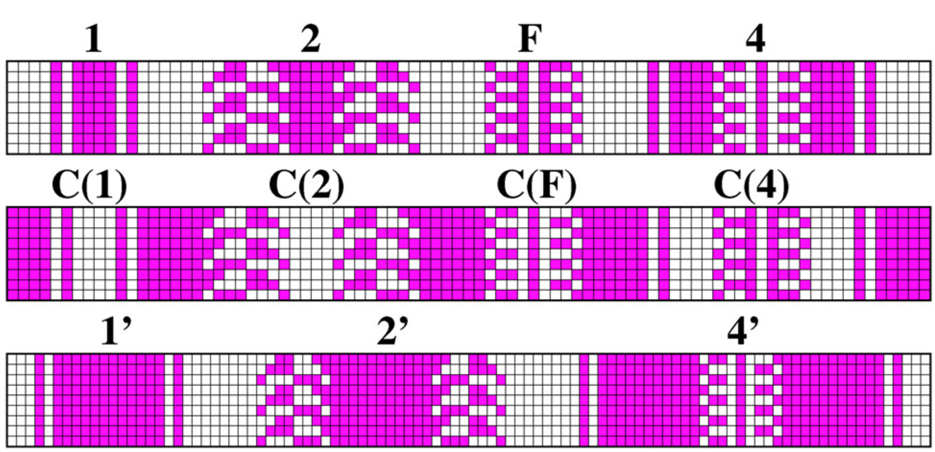

Fig. 4. Gliders arising when interfaces are glued together. Top: the simplest motifs surviving on a white background representing zeros. Middle: conjugates of the gliders in the top row, white structures surviving on a black background of ones. Bottom: "fat" gliders, i.e. the same gliders seen on the top row but now with "inflated" inner cores of ones. Note the different possibilities of combining interfaces.

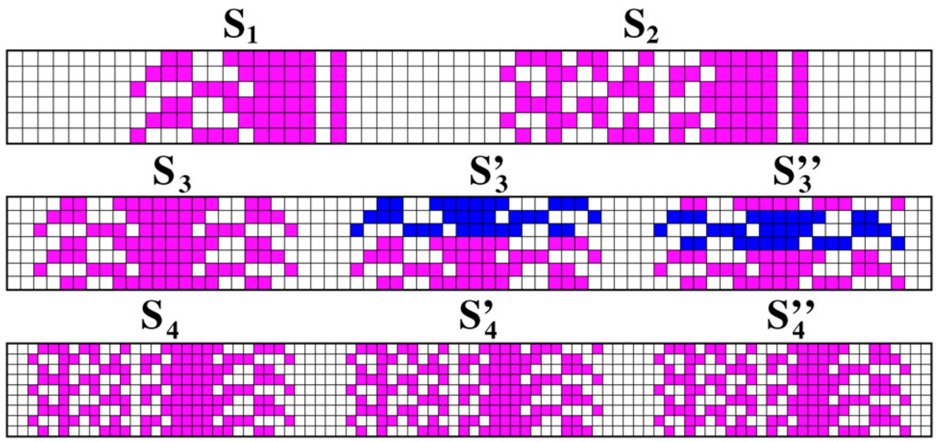

Fig. 5. Top: Static hybrid gliders formed by juxtaposing distinct elementary interfaces. The four black sites separating interfaces may be inflated arbitrarily, as indicated in Fig. 4. Center: static gliders formed by dephasings due to the different ways of juxtaposing identical period-3 interfaces. Note that S_3 has "rotational symmetry" with respect to its central axis while S_3' and S_3'' have "helicoidal symmetry", i.e. involve a reflection plus a one time-step shift as hinted by the additional coloring. The minimum distance between interfaces is larger for leftmost gliders, to prevent white cells from interacting. Bottom: Hybrid gliders due to the three possible time-dephasings between different period-3 interfaces. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

ering structures as moving on backgrounds different than those originally intended. For instance, cutting both gliders 1 and 2 along their fat inner cores one obtains the conjugate of glider S_1, shown in Fig. 5, etc. More generally, by removing all labelings from Fig. 4 one produces optical illusions, with different people recognizing different gliders, at first.

Fig. 5 illustrates typical static gliders obtained when juxtaposing interfaces which may be similar or not. As the figure shows, a number of closely related patterns arises when combining interfaces having temporal periodicities larger then 1. For instance, with the interfaces C and $\overline{C}$ we may generate three distinct but closely related fingers, denoted S_3, S_3', S_3'' in Fig. 5. While S_3 has "rotational symmetry" with respect to its central axis, its partners S_3' and S_3'' have a more elaborate "helicoidal symmetry" involving a spatial reflection plus a one time-step shift as may be recognized comparing their periods painted with the darker coloring. Further, note that the minimum distance between the pair of interfaces forming S_3 and S_4 is larger than that of their partners in order to prevent cells from interacting.

How complex is rule 52? Since all computable asymptotic dynamics of rule 52 was found to be producible by juxtaposing no more than seven elementary interfaces, it seems hard to argue its intrinsic complexity to be very high, according to any of the usual measures of complexity, particularly that organized around the symbolic dynamics of stationary symbol sequences [1]. We do not see how rule 52 could support universal computation. In fact, our results seem to support doubts about the very existence of complex cellular automata [31]. While of course a small number of interfaces might prove insufficient to generate all possible gliders when one lets the system size to grow without bound (thermodynamic limit), a systematic multispin-coding search conducted for very large lattice sizes [30] has not revealed new interfaces. In fact, the interface labeled E in Fig. 3, the largest interface found, was obtained not while probing the thermodynamic limit with random initial conditions but by symmetry considerations, indicating that the symmetries underlying interfaces may be fruitfully exploited to generate them. Further, for small lattice sizes it is possible to find even greater regularity, like the space-filing tilings observed for rule 20 very recently [29].

It seems worth emphasizing that although the asymptotic behavior in the thermodynamic limit discussed here is a deep and enticing question from a conceptual point of view, there are many applications of great practical importance which may surely profit from the short-lived complexity exhibited by rule 52 during the first few generations and for lattices of moderately small sizes, particularly for problems in socio-physics, biophysics, or in problems involving excitable media [32], specially when allowing for more local degrees of freedom. Interesting open questions for systems operating under rules similar [30] to rule 52 are (i) how to harness short-time complexity to locate expeditiously the future positioning of interfaces and gliders along the lattice, and (ii) what sort of strongly selective mechanism is responsible for so effectively reducing periods and motifs which survive. For, it is remarkable that such huge phase-spaces support only a very limited gamut of complexity.

In conclusion, we briefly mention that the interfaces reported here may be efficiently studied with a *spatial updating algorithm* [33], a tool that allows one to prove that the static interfaces found empirically here are the only interfaces possible with spatial periods up to 10, independently of the size of the automata. The spatial updating algorithm provides an alternative way to determine the dynamics of automata of *arbitrary size*, a way of taking into account the complexity of the connections in the lattice [33].

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